A Numerical Observation on the Inverse Fine-Structure Constant

Connections with a perturbative series for m_p/m_e

Author : Judicael Brindel, Independent Researcher, Cotonou, Benin

ORCID: 0009-0007-4590-9874

February 2026

Abstract

We report a numerical observation concerning the inverse fine-structure constant 1/α. We find that the following relation holds to within the experimental uncertainty of CODATA 2018:

$$\mathbf{9/\alpha + (35/2\pi)\alpha^3 = 236 \times \ln(6\pi^3)}$$

The residual error is 1.3×10^{-9}, within the CODATA 2018 experimental uncertainty on α. The coefficients 236, 9, 35, and the arguments 6 and π^3 share structural elements with a previously reported perturbative series for m_p/m_e [1]. No theoretical derivation is offered.

1. Introduction

The fine-structure constant $\alpha \approx 1/137.036$ is one of the fundamental dimensionless constants of physics. Its value is known experimentally to high precision (CODATA 2018 [2]) but has no derivation from first principles within the Standard Model.

Numerous empirical approximations to 1/α have been proposed in the literature. The present note reports a new numerical observation of the form $9/\alpha + (35/2\pi)\alpha^3 = 236 \times \ln(6\pi^3)$, and notes its structural connection with a perturbative series for the proton-to-electron mass ratio m_p/m_e reported in [1].

2. The Numerical Observation

We report the following relation:

$$\mathbf{9/\alpha + (35/2\pi)\alpha^3 = 236 \times \ln(6\pi^3)}$$

Using CODATA 2018 values $\alpha = 7.2973525693(11) \times 10^{-3}$ [2]:

Left-hand side	Right-hand side	Residual
$9/\alpha + (35/2\pi)\alpha^3$	$236 \times \ln(6\pi^3)$	1.3×10^{-9}
1233.323993918	1233.323993919	< exp. uncertainty

The CODATA 2018 uncertainty on α propagates to an uncertainty of 1.9×10^{-7} on the left-hand side. The residual 1.3×10^{-9} is well within this bound.

At leading order (neglecting the α^3 term), this simplifies to:

$$\mathbf{1/\alpha \approx (236/9) \times \ln(6\pi^3)}$$

with a relative error of 1.8×10^{-9} (also within experimental precision).

3. Structure of the Coefficients

The coefficients appearing in the formula are:

$$9 = 3^2,\ 236 = 4 \times 59,\ 35 = 5 \times 7$$

The factor $9 = 3^2$ is notable: it appears on both sides of the equation when written in the form $9/\alpha + (35/2\pi)\alpha^3 = 236 \times \ln(6\pi^3)$. Numerically, $9/\alpha \approx 1233.3$, which is within 1.3 units of the mass of the Delta(1232) baryon (1232 MeV) in units of m_e. This numerical coincidence is noted without theoretical interpretation.

The argument $6\pi^3$ of the logarithm shares the factor 6 with the leading term $6\pi^5$ of the perturbative series for m_p/m_e reported in [1]. The power π^3 also appears in the ratio $c_3/c_2 = \pi^3/4$ of the coefficients of that series.

4. Connection with the Series for m_p/m_e

The perturbative series for m_p/m_e reported in [1] reads:

$$\mathbf{m_p/m_e = 6\pi^5 + (3\pi/2)\alpha + \pi\alpha^2 + (\pi^4/4)\alpha^3 + \cdots}$$

That series can be rewritten as $m_p/m_e = 6\pi^5 + \pi\alpha \cdot h(\alpha)$, where $h(\alpha) = 3/2 + \alpha + (\pi^3/4)\alpha^2$ and $h(0) = 3/2$.

The present observation shares several structural elements with [1]:

(1) Both formulas involve the factor 6 and powers of π.

(2) The power π^3 appears in $h(\alpha)$ and in $\ln(6\pi^3)$.

(3) Both formulas include a correction term of order α^3.

(4) The factor $9 = 3^2$ connects to $h(0) = 3/2 = 3/2$ (the spin of the Delta(1232) baryon) noted in [1].

Whether these shared elements reflect a deeper common structure or are numerical coincidences remains unknown. The two formulas are algebraically independent: substituting the leading-order approximation $\alpha \approx 9/(236 \times \ln(6\pi^3))$ into the series for m_p/m_e reproduces the CODATA value to within its experimental uncertainty, but this does not constitute a derivation of either formula from the other.

5. Conclusion

We have reported the numerical observation:

$$\mathbf{9/\alpha + (35/2\pi)\alpha^3 = 236 \times \ln(6\pi^3)}$$

with a residual of 1.3×10^{-9}, within the CODATA 2018 experimental uncertainty. The structural elements shared with the perturbative series for m_p/m_e [1] — the factor 6, the power π^3, the term in α^3, and the factor $9 = 3^2$ — suggest that both observations may reflect a common underlying structure. A theoretical derivation from QED, QCD, or a more fundamental framework would be required to establish this connection.

References

[1] J. Brindel, "A Perturbative Series for the Proton-to-Electron Mass Ratio," Zenodo (2026). https://doi.org/10.5281/zenodo.18693489

[2] CODATA 2018: $\alpha = 7.2973525693(11) \times 10^{-3}$, $\alpha^{-1} = 137.035999084(21)$.

This research was conducted with computational assistance from Claude (Anthropic) for the algorithmic exploration and formal verification.